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MODELING OF EIGENOSCILLATIONS COUPLED DETUNED DIELECTRIC RESONATORS[1]

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МОДЕЛЮВАННЯ ВЛАСНИХ КОЛИВАНЬ ЗВ'ЯЗАНИХ РОЗСТРОЄНИХ ДІЕЛЕКТРИЧНИХ РЕЗОНАТОРІВ

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Introduction

While attempting to use DRs in the millimeter or shorter wavelength ranges, the accurate tuning problem arises frequently in order to receive necessary scattering parameters. Herewith obtaining the rigorously equal DRs is usually a very difficult task [1 - 4], therefore the investigation of DR properties makes sense in cases when their dimensions differ. The understanding of the interaction processes in complex DR systems becomes possible while providing the proper simulation of their properties.

Eigenmode simulation of detuned DRs

Suppose that the eigenoscillation field $(\vec{e}_s, \vec{h}_s)$ as well as the resonance frequencies ω_s ($s = 1, 2, ..., N$) of each DR are known. We assume that all high-Q DR fields vary with frequency by known law $\alpha_s(\omega)$, but their spatial distribution remains the same:

$$\begin{bmatrix} \vec{e}_s(\omega, \vec{r}) \\ \vec{h}_s(\omega, \vec{r}) \end{bmatrix} \simeq \alpha_s(\omega) \begin{bmatrix} \vec{e}_s(\vec{r}) \\ \vec{h}_s(\vec{r}) \end{bmatrix} \text{ for } \omega \simeq \omega_s . \quad (1)$$

Herewith we suppose that

$$\alpha_s(\omega_s) = 1; \ \lim_{|\omega - \omega_s| \to \infty} \alpha_s(\omega) = 0 . \quad (2)$$

The expansion coefficients of the DR field (1) on the propagation wave field of the line $(\vec{E}_\nu^{\pm}, \vec{H}_\nu^{\pm})$ are defined by applying the DR surface integrals:

$$c_\nu^{t\pm}(\omega) = -(1/2) \oint_{S_t} \left\{ \left[\vec{e}_t(\omega), \vec{n} \right] (\vec{H}_\nu^{\pm})^* + \left[\vec{n}, \vec{\mathrm{h}}_t(\omega) \right] (\vec{\mathrm{E}}_\nu^{\pm})^* \right\} ds .$$

For the non propagating waves

[1] http://radap.kpi.ua/radiotechnique/article/view/1165

$$\begin{pmatrix} c_\nu^{t\pm}(\omega) \\ d_\nu^{t\pm}(\omega) \end{pmatrix} = \begin{pmatrix} - \\ + \end{pmatrix} \mathrm{i}/2 \cdot \oint_{s_t} \left\{ \left[\vec{e}_t(\omega), \vec{n} \right] (\vec{H}_\nu^{\mp})^* + \left[\vec{n}, \vec{\mathrm{h}}_t(\omega) \right] (\vec{\mathrm{E}}_\nu^{\mp})^* \right\} ds.$$

Here, for compactness, the dependence on spatial coordinates $\vec{r}$ has been omitted.

Then, using (1), we obtained:

$$c_\nu^{t\pm}(\omega) = \alpha_t(\omega) c_\nu^{t\pm}; \quad d_\nu^{t\pm}(\omega) = \alpha_t(\omega) d_\nu^{t\pm}, \tag{3}$$

where

$$c_\nu^{t\pm} = -1/2 \oint_{s_t} \left\{ \left[\vec{e}_t, \vec{n} \right] (\vec{H}_\nu^{\pm})^* + \left[\vec{n}, \vec{\mathrm{h}}_t \right] (\vec{\mathrm{E}}_\nu^{\pm})^* \right\} ds;$$

$$\begin{pmatrix} c_\nu^{t\pm} \\ d_\nu^{t\pm} \end{pmatrix} = \begin{pmatrix} - \\ + \end{pmatrix} \mathrm{i}/2 \oint_{s_t} \left\{ \left[\vec{e}_t, \vec{n} \right] (\vec{H}_\nu^{\mp})^* + \left[\vec{n}, \vec{\mathrm{h}}_t \right] (\vec{\mathrm{E}}_\nu^{\mp})^* \right\} ds \tag{4}$$

are known expansion coefficients [5] at the resonance frequency ω_t for the propagating and non propagating waveguide waves, respectively.

The energy, stored inside t-th DR at frequency ω can be defined as:

$$w_t(\omega) = 1/4 \int_{v_t} \left[\varepsilon_1 \left| \vec{e}_t(\omega) \right|^2 + \mu_0 \left| \vec{h}_t(\omega) \right|^2 \right] dv,$$

($t = 1, 2, ..., N$). From (1) the following expression can be obtained:

$$w_t(\omega) = \left| \alpha_s(\omega) \right|^2 w_t, \tag{5}$$

where $$w_t = 1/4 \int_{v_t} \left[\varepsilon_1 \left| \vec{e}_t \right|^2 + \mu_0 \left| \vec{h}_s \right|^2 \right] dv.$$

The mutual coupling coefficients of the s-th and the t-th DRs on the damped and expanding waves at frequency ω can be defined as:

$$k_{st}(\omega) = \frac{-1}{\omega_t w_t} \sum_{n=1}^{\infty} \left[c_n^s(\omega) c_n^t(\omega)^* e^{-\Gamma \Delta z_{st}} - d_n^s(\omega) \mathrm{d}_n^t(\omega)^* e^{-\Gamma \Delta z_{st}} \right],$$

$$\tilde{k}_{st}(\omega) = \frac{1}{\omega_t w_t} c_e^s(\omega) c_e^t(\omega)^* e^{-i\Gamma \Delta z_{st}}. \tag{6}$$

By using (3), (5), the following expression can be obtained

$$k_{st}(\omega) = \frac{\alpha_s(\omega)}{\alpha_t(\omega)} k_{st}; \quad \tilde{k}_{st}(\omega) = \frac{\alpha_s(\omega)}{\alpha_t(\omega)} \tilde{k}_{st}, \tag{7}$$

where, similarly

$$k_{st} = \frac{-1}{\omega_t w_t} \sum_{n=1}^{\infty} \left[c_n^s c_n^{t*} e^{-\Gamma \Delta z_{st}} - d_n^s \mathrm{d}_n^{t*} e^{-\Gamma \Delta z_{st}} \right]; \ \tilde{k}_{st} = \frac{1}{\omega_t w_t} c_e^s c_e^{t*} e^{-i\Gamma \Delta z_{st}}. \tag{8}$$

As follows from (7):

$$\tilde{k}_{tt}(\omega) = \tilde{k}_{tt} = \tilde{k}_t. \tag{9}$$

Here Γ - is the guided wavelengths; z_s - is the longitudinal coordinate of the s-th DR in the transmission line; $\Delta z_{st} = \left| z_s - z_t \right|$.

The eigenmode problem solution of the N-DR system $(\vec{e}(\omega),\vec{h}(\omega))$ also can be found as a superposition of fields of the isolated resonators $(\vec{e}_s(\omega),\vec{h}_s(\omega))$ (1), $(s=1,2,...,N)$:

$$\vec{e}(\omega)=\sum_{s=1}^{N} b_s\vec{e}_s(\omega)\,;\;\vec{h}(\omega)=\sum_{s=1}^{N} b_s\vec{h}_s(\omega)\,. \qquad (10)$$

Here b_s - is the unknown complex amplitude of the s-th resonator mode. The expansion coefficients b_s as well as the complex eigenmode DR-system frequencies ω can be generally obtained from Maxwell's equations, using perturbation theory [5]. After simple transformations of the eigenmode fields $(\vec{e}(\omega),\vec{h}(\omega))$ and $(\vec{e}_s(\omega),\vec{h}_s(\omega))$ all values of b_s can be found that satisfy the equation system:

$$\sum_{s\neq t}^{N}[\mathrm{k}_{st}(\omega_t)+\mathrm{i}\tilde{\mathrm{k}}_{st}(\omega_t)]b_s+[i\tilde{k}_{tt}(\omega)-\lambda_t]b_t=0\,,\quad (t=1,2,...,N\,),$$

By using (2), (9), the following can be obtained:

$$\sum_{s\neq t}^{N}\alpha_s(\omega_t)\kappa_{st}b_s+(i\tilde{k}_t-\lambda_t)b_t=0\,, \qquad (11)$$

where

$$\lambda_t=2\cdot(\frac{\omega-\omega_t}{\omega_t})\,,\qquad (t=1,2,...,N); \qquad (12)$$

$\kappa_{st}=\mathrm{k}_{st}+\mathrm{i}\tilde{\mathrm{k}}_{st}$; ω_t - is the real part of frequency of t-th DR mode.

By equating the determinant of (11) to zero, the complex natural frequencies $\omega=(\omega^1,\omega^2,...,\omega^N)$ of the coupled DR system can be found.

The quality factor of the DR system can be derived by applying the known formula: $Q^s=\mathrm{Re}[\omega^s]/2\,\mathrm{Im}[\omega^s]$, $(s=1,2,...,N)$

If $|\omega_t-\omega_s|\gg\omega_s$ for every s, then, as follows from (2), (11), the t-th DR oscillations are independent from other resonators: $\lambda_t=i\tilde{k}_t$ and $\mathrm{Re}(\omega)=\omega_t$; $\mathrm{Im}(\omega)=1/2\omega_t\tilde{k}_t$; the quality factor of the t-th mode will be: $Q^t=1/\tilde{k}_t$.

The unknown "amplitude" functions $\alpha_s(\omega)$ are assumed to vary as in the case of scattering by the s-th isolated DR:

$$\alpha_s(\omega)=\frac{1+\tilde{k}_sQ_s^D}{Q_s(\omega)}\,; \qquad (13)$$

where $\qquad Q_s(\omega)=\omega/\omega_s+2iQ_s^D(\omega/\omega_s-1-i/2\tilde{k}_s)$;

$Q_s^D=1/tg\delta_s$, $tg\delta_s$ - is the dielectric loss tangent of the s-th resonator. For the case $Q_s^D=10^3$; $k_s=0{,}01$ the functions $\alpha_s(\omega)$ have shown in Fig. (1), a.

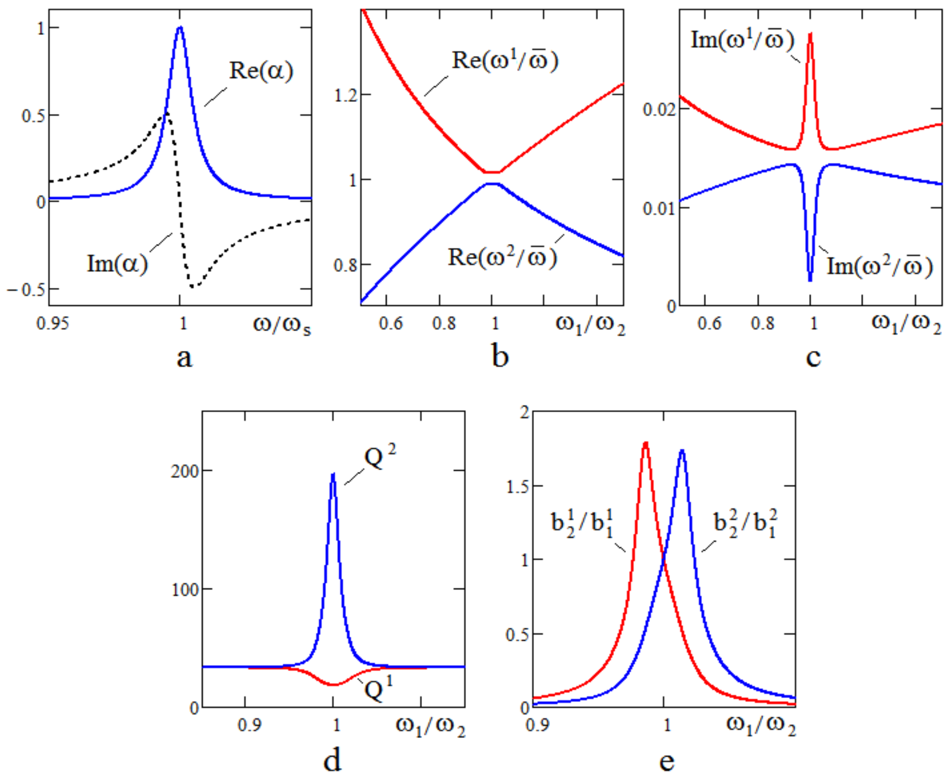

Fig.1. Amplitude of one DR as a function of frequency (a). The frequencies of two coupled oscillations of DRs ($\bar{\omega}=\sqrt{\omega_1\omega_2}$) as a function of the relative frequencies of isolated DRs (b, c). Q-factors of coupled oscillations (d). Relative amplitudes of two detuned DRs (e). $\tilde{k}_1=\tilde{k}_2=0,03$; $\kappa_{12}=\kappa_{21}=0,03+i0,025$; $Q_1^D=Q_2^D=10^3$

The expressions, obtained for the coupling coefficients, are not free from controversy. For example, the mutual coupled coefficients should be calculated at various frequencies at the same time, because the ratio (8) contains coefficients $c_n^{s\pm};c_n^{t\pm}$ for different detuned DRs. The inconsistency can be avoided by remembering the finiteness of the frequency distribution of the field of each resonator ((2); Fig. (1), a). Interaction of high-Q resonators becomes noticeable only on close frequencies, so in this case it is sufficient to calculate the coupling coefficients only for $\omega_s=\omega_t$.

Several out-of-tune DR coupled oscillations

Equations (11) allow obtaining frequencies of coupled oscillations of two DRs in simpler way:

$$\frac{\omega^{1,2}}{\sqrt{\omega_1\omega_2}}=\frac{1}{4}[\sqrt{\frac{\omega_1}{\omega_2}}(i\tilde{k}_1+2)+\sqrt{\frac{\omega_2}{\omega_1}}(i\tilde{k}_2+2)\pm\sqrt{\Delta}]$$

$$\Delta=[\sqrt{\frac{\omega_1}{\omega_2}}(i\tilde{k}_1+2)+\sqrt{\frac{\omega_2}{\omega_1}}(i\tilde{k}_2+2)]^2+4[\alpha_1(\omega_2)\alpha_2(\omega_1)\kappa_{12}\kappa_{21}-(i\tilde{k}_1+2)(i\tilde{k}_2+2)]$$

$$b_1=\lambda_2-i\tilde{k}_2;\;\; b_2=\alpha_1(\omega_2)\kappa_{12}. \qquad (14)$$

In Fig. (1) the coupled oscillation parameters of two detuned DRs have been shown. One can see, that the DR frequency detuning has been conducted for the cases of all structure parameters’ variation. The real parts of coupled resonance frequencies never intersect each other, if both real parts of mutual coupling coefficients are no equal to zero. The quality factors reach their extreme values for equal DRs only (Fig. (1), d). The relative amplitudes of the coupled oscillations are equal to each other when $\omega_1 = \omega_2$ for the equal DRs only (e).

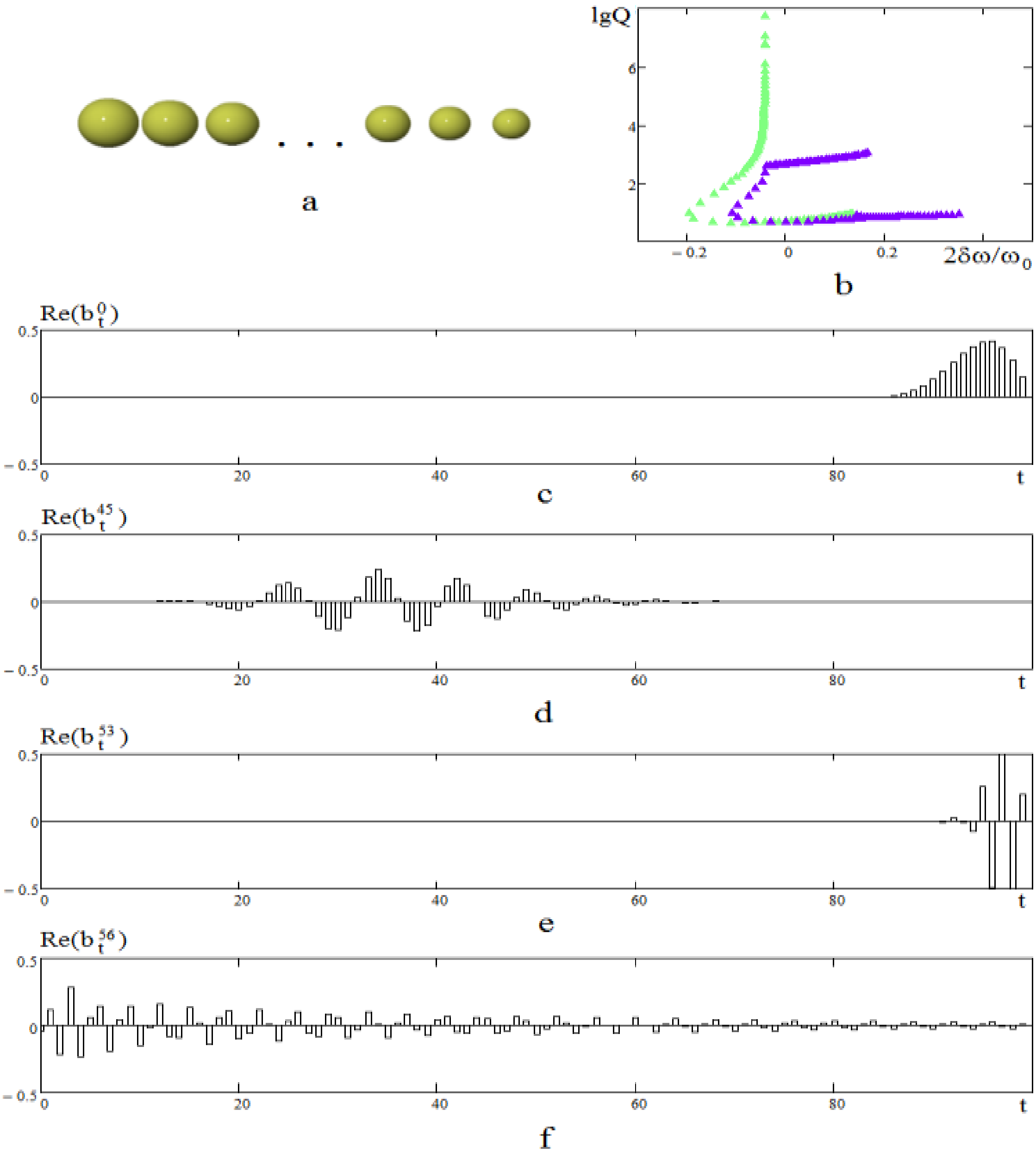

Fig. 2. One-dimensional lattice of different Spherical DRs (a). The DR radii decreased by linear law: $r_s = (1 - 10^{-3}s)r_0$, where r_0 - is the radius of the first DR. Relative frequencies and Q-factors of the coupling oscillations of the lattices consisting of 100 DRs (b); for identical DRs (green points); for different DRs (blue points) (b). Amplitude allocation of the most representative oscillations; $\varepsilon_{1r} = 16$; $Q_s^D = 10^3$; $f_0 = 300$ THz.

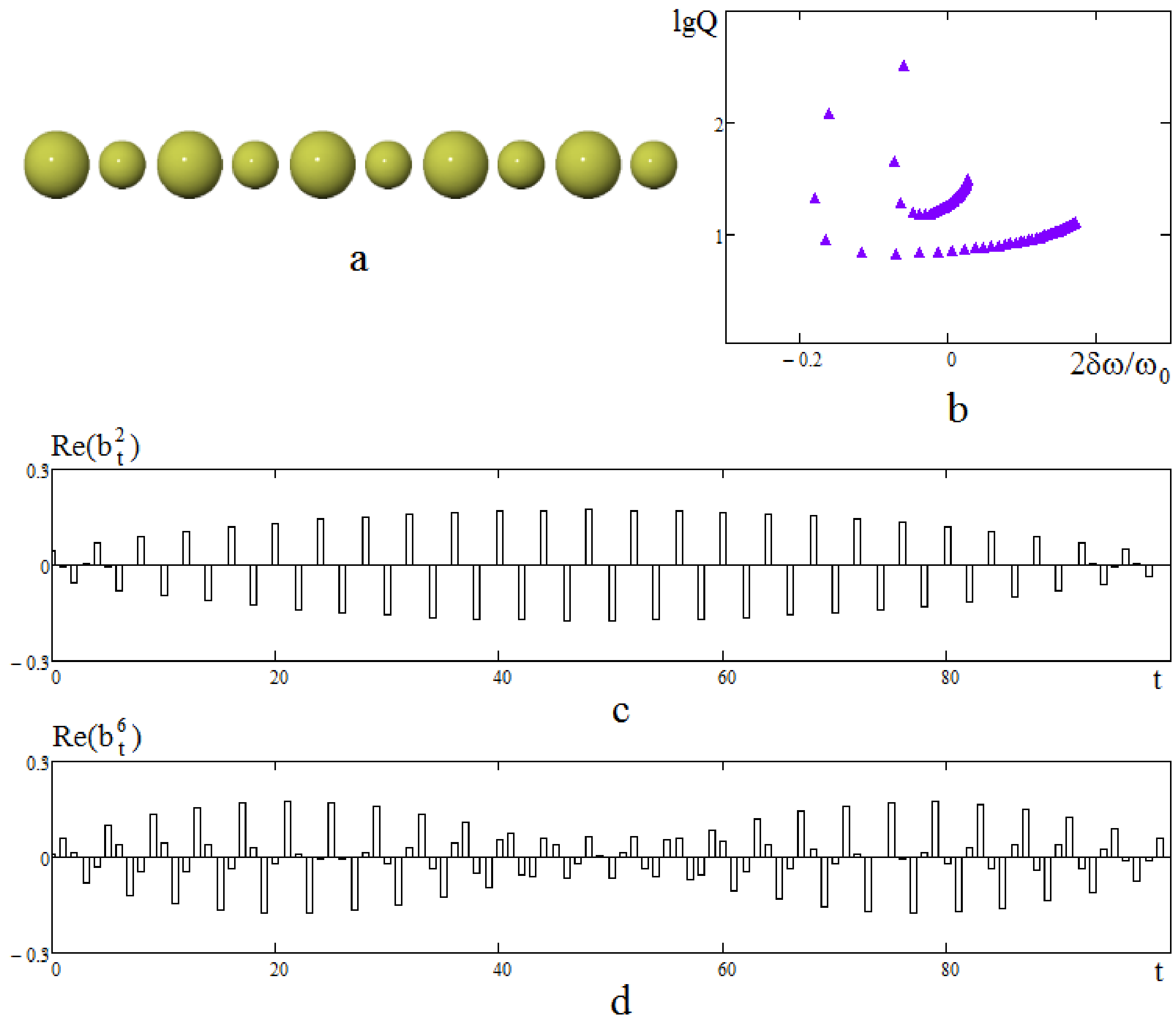

Fig. 3. Two embedded one-dimensional lattices of different Spherical DRs (a). The DR radii of the first lattice make up r_0, of the second lattice $-\ 0,95r_0$. The relative frequencies and quality factors of coupling oscillations of the lattices of 100 DRs (b). Amplitude allocation of the most high-Q oscillations (c, d); $\varepsilon_{1r}=16$; $Q_s^D=10^3$; $f_0=300$ THz.

Coupled resonances of the Detuned Spherical DRs' Lattices in the Open Space

Using the system of equations (11), the lattices consisting of a large number of the Spherical DRs have been investigated. Let's suppose that all resonators are excited in the basic magnetic mode H_{111} [5]. The magnetic field in all DR centers has been oriented orthogonally to the lattice's axes in one-dimensional case (Fig. (2), (3), a) and orthogonally to the lattice plane for two-dimensional case (Fig. (4) - (6), a). All coupling coefficients (8 - 9) have been calculated based on [6].

In the cases when the DR dimensions are changed smoothly (Fig. (2), (4), (5), a) some coupled resonances have demonstrated shape allocated oscillations (Fig. (2), (4), (5), c - e), occupying certain segment of the lattice only. This phenomenon, evidently, can be connected with various DR dimensions, located in the specified part of the lattice that define frequencies of the coupling oscillations.

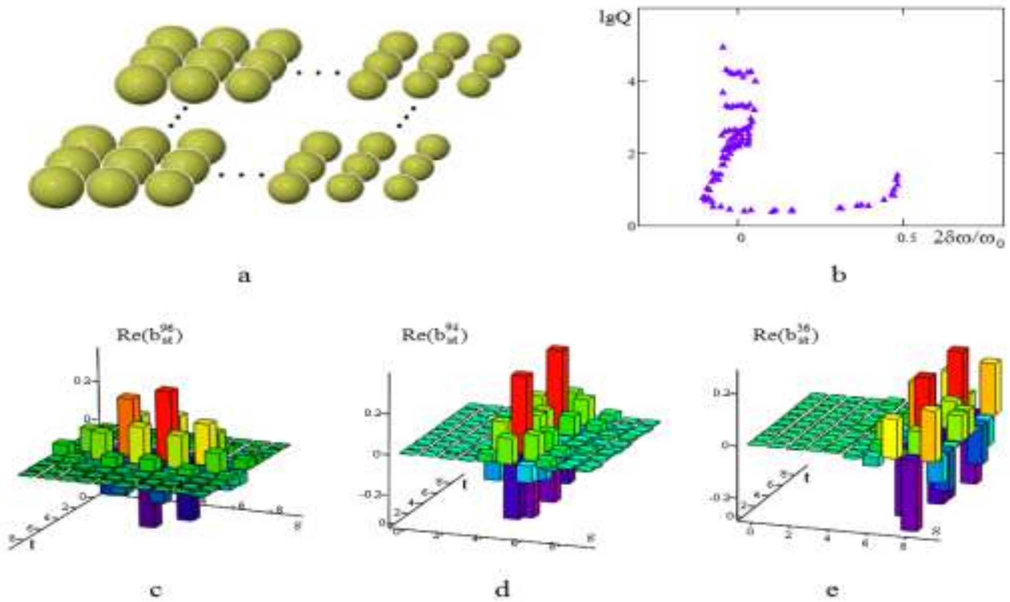

Fig. 4. Square lattice of the detuned Spherical DRs ($\varepsilon_{1r}=16$; $Q_s^D=10^3$; the resonance frequency of DRs in the first row is $f_0=300$ THz) in the Open Space (a). The DR radii of the posterior rows decrease by linear law: $r_s=(1-5\cdot10^{-3}s)r_0$, where r_0 - is the radius of the first DR. Q-factors and frequencies of the lattice (b). The typical localized DR's amplitude distributions (c - e).

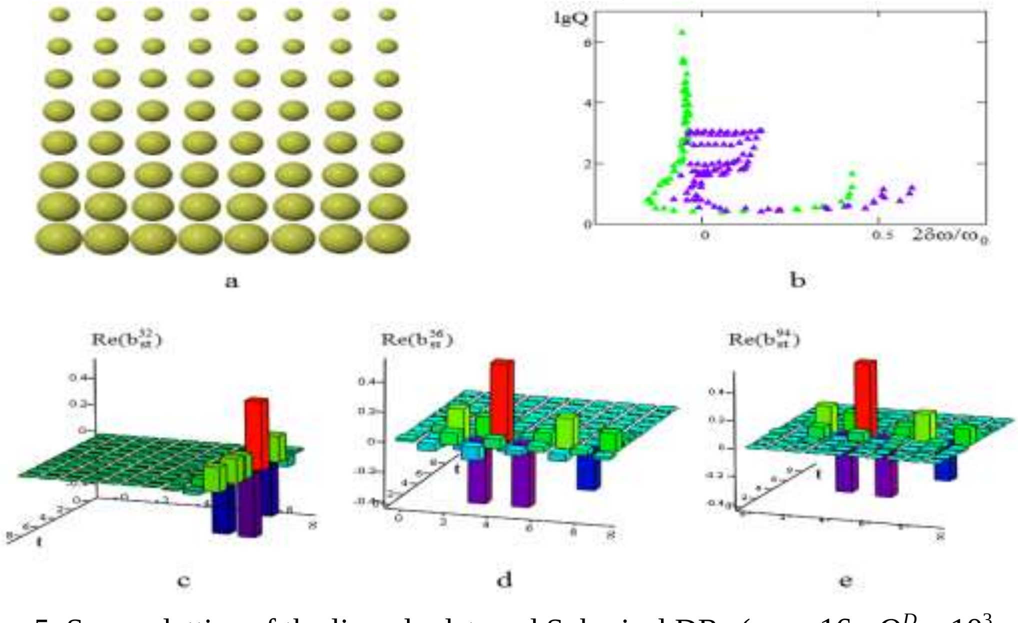

Fig. 5. Square lattice of the linearly detuned Spherical DRs ($\varepsilon_{1r}=16$; $Q_s^D=10^3$; the resonance frequency of first DRs is $f_0=300$ THz) in the Open Space (a). The DR radii decrease by linear law: $r_s=(1-10^{-3}s)r_0$ ($s=1...100$), where r_0 - is the radius of the first DR. Q-factors and frequencies of the identical DRs lattice (green points); linearly detuned DR's (blue points) (b). The amplitude distributions of typically localized DR (c - e).

If the structure was composed of two different kinds of DRs (Fig. (3), (6), (7), a), in some cases their frequencies and quality factors would be allocated as in case of two different lattices (Fig. (3), (7), b). Herewith the amplitudes' allocation of the most high-Q oscillations has form of the antiphased distributions (Fig. (3), c - d; Fig. (6), c - d; Fig. (7), d - e)

In all cases the DR detuning results in the Q-factor decreasing for all oscillations of the lattice. For example, the Fig. (2), (5), (6), b show Q-factors and the frequencies of the lattice, consisting of equal DRs, labeled by green points, and by contrast the Q-factors and the frequencies of the lattice of DRs, having the same material, but consisting of detuned DRs, have been illustrated by the blue points. Obviously, the maximal Q-factor of the lattice have been decreased from $10^8 - 10^6$ to $10^6 - 10^3$, respectively.

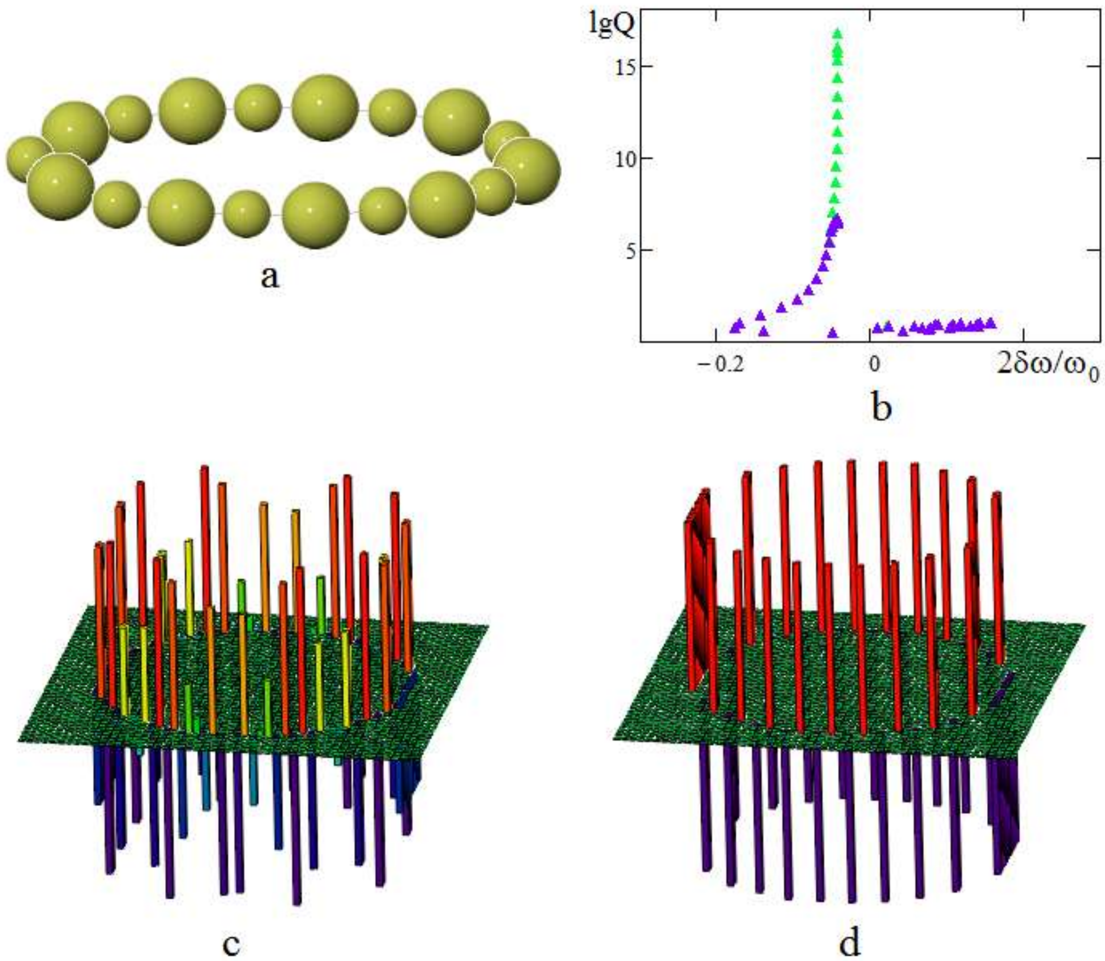

Fig. 6. Two embedded Ring lattices of the Spherical DRs ($\varepsilon_{1r} = 16$; $Q_s^D = 10^3$; the first DRs' resonance frequency is $f_0 = 300$ THz) in the Open Space (a). The radii of the first DRs are r_0; the radii of the second DRs of the lattice are $(1 - 0{,}0001)r_0$. Q-factors and frequencies of the identical DR lattice (green points); the embedded Ring DR lattices (blue points) (b). Amplitude allocation of some high-Q oscillations (c - d).

Conclusions

By using the perturbation theory for the equation system, the amplitudes and complex frequencies of the detuned coupled DRs have been obtained.

The study shows that the DRs detuning has been accompanied by decrease in maximal Q-factor of coupled oscillations.

If the DR dimensions are changed by the smooth law, the certain coupled resonances can be allocated only in a part of the lattice.

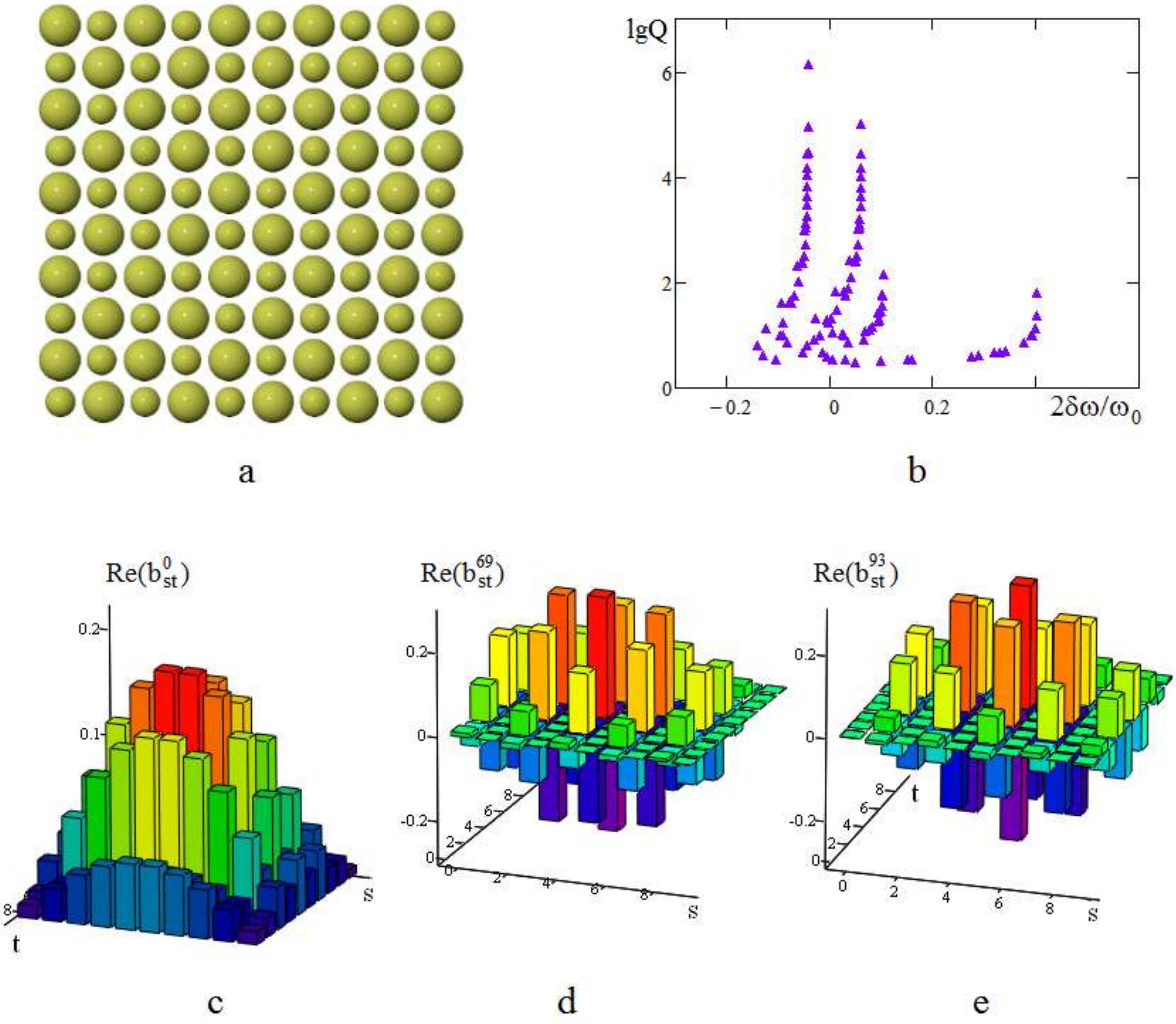

Fig. 7. Two embedded Square lattices of the Spherical DRs ($\varepsilon_{1r} = 16$; $Q_s^D = 10^3$; first DRs resonance frequency is $f_0 = 300$ THz) in the Open Space (a). The radii of the first lattice DRs are r_0; radii of the second DRs are $0{,}95r_0$. Q-factors and frequencies of the lattice (b). Amplitude allocation of most high-Q oscillations (c - e).

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Трубін О. О. ***Моделювання власних коливань зв'язаних розстроєних діелектричних резонаторів.*** *Запропоновано електродинамічна модель розстроєних діелектричних резонаторів. Одержана система рівнянь, зв'язуючи частоті та амплітуди резонаторів. Досліджуються закони зв'язаних коливань двох різних діелектричних резонаторів. Розглянуто власні коливання одно-, двох-вимірних решіток, складених із різних сферичних ДР у відкритому просторі. Відзначено нові властивості решіток різних ДР.*

Трубин А. А. ***Моделирование собственных колебаний расстроенных диэлектрических резонаторов.*** *Рассматривается система связанных диэлектрических резонаторов различных размеров, расстроенных по частоте. На основе теории возмущений, построена простая аналитическая модель связанных колебаний. Получена система уравнений, связывающая между собой частоты и амплитуды резонаторов. Рассмотрены основные закономерности изменения параметров системы двух различных ДР при вариации относительной расстройки их резонансных частот. Исследованы одно- и двух-мерные решетки сферических ДР различных размеров. Показано, что изменение относительных размеров резонаторов в структуре решетки как правило сопровождается заметным уменьшением добротности их связанных колебаний. Установлено появление локализованных в пространстве решетки связанных колебаний, возникающее при плавном изменении размеров резонаторов. Показано, что дальнейшее изменение относительных размеров ДР приводит к их независимым собственным колебаниям.*

Trubin A. A. ***Modeling of eigenoscillations coupled detuned Dielectric Resonators .*** *An electrodynamic model of the detuned Dielectric Resonators' (DRs) system has been proposed. A new equation system connecting frequencies and resonator' amplitudes has been obtained. The Coupled oscillations law of two different DRs has been explored. The resonances of one-, two-dimensional lattices, composed of different Spherical DRs in the Open Space have been considered. New properties of the detuned DR Lattices have been discovered.*

Keywords: *detuned dielectric resonators, coupled oscillations, equation system, lattice.*